Quiz 6

Name: _Solution Key_

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. Find the radius and interval of convergence of the power series

$$\sum_{k=0}^{\infty} \frac{(-\pi)^k}{2^{2k+1}}(x-1)^k.$$

Root Test:

$$\lim_{k \to \infty} \left(\left| \frac{(-\pi)^k}{2^{2k+1}}(x-1)^k \right| \right)^{\frac{1}{k}} = \lim_{k \to \infty} \frac{\pi}{2^{2+\frac{1}{k}}}|x-1| = \frac{\pi}{4}|x-1| < 1$$

$$|x-1| < \frac{4}{\pi}.$$

$$R = \frac{4}{\pi}.$$

Endpoints: $x = 1 + \frac{4}{\pi}$

$$\sum_{k=0}^{\infty} \frac{(-\pi)^k}{4^k \cdot 2} \frac{4^k}{\pi^k} = \sum_{k=0}^{\infty} \frac{1}{2}(-1)^k \quad \text{diverges by divergence test}$$

$$x = 1 - \frac{4}{\pi}$$

$$\sum_{k=0}^{\infty} \frac{(-4)^k}{\pi^k} \frac{\pi^k}{4^k \cdot 2} = \sum_{k=0}^{\infty} \frac{1}{2} \quad \text{diverges by divergence test}$$

So I.O.C is $\left(-\frac{4}{\pi}+1, \frac{4}{\pi}+1 \right)$.